Module 10: Control System for Electric and Hybrid Electric Vehicles

Lecture 35: Control Systems for the HEV and EVs

Control Systems for the HEV and EVs

Introduction

The topics covered in this chapter are as follows:

- Function of Control System in HEVs and EVs
- Elementary of Control Theory
- Overview of Control System: The Electronic Control Unit (ECU)
- Control Area Network
- Control Variables

Function of the Control System in HEVs and EVs

The major functions of the control system are:

i. to maximize the fuel efficiency
ii. to minimize the exhaust emissions.

The fuel efficiency and emissions are mutually conflicting and some of the reasons why this happens are:

i. when more energy is extracted by the ICE (thereby increasing the ICE efficiency) the exhaust temperature goes down. At lower temperatures, the chemical reactions associated with the combustion of unburned hydrocarbons may not occur.
ii. Increase in compression ratio, which enhances fuel economy, also raises temperature in ICE. Increased temperature increases both CO and oxides of nitrogen represented by NOx.

The minor functions of the control system are component monitoring and protection such as:

i. battery state of charge (SOC) monitoring
ii. Battery temperature monitoring
iii. EM overheating
iv. ICE overheating

The battery merits special attention to avoid failure and to assure long life. The control system generally provides a fail safe mode in the event of failures. This gives a limp-home capability that helps the manufacturer to retain some credibility with the customers. One of the functions of control systems, which is minor in cost but valuable in practice, is onboard diagnostics (OBD).

In **Figure 1**, a simple HEV drivetrain is shown. For the HEV drivetrain shown in **Figure 1**, the complexity of the control system used in HEV may be appreciated by considering the various operational modes (**Table 1)** and the interplay between many components. The switching from one mode to another must be as smooth. As the HEV's operation mode changes, several parameters of the ICE that have to be controlled are:

i. Ignition timing
ii. Tuned intake manifold
iii. Camshaft angle for exhaust valves
iv. Camshaft angle for intake valves
v. Fuel injector settings which includes
    a. Timing of injection
    b. Fuel flow rate

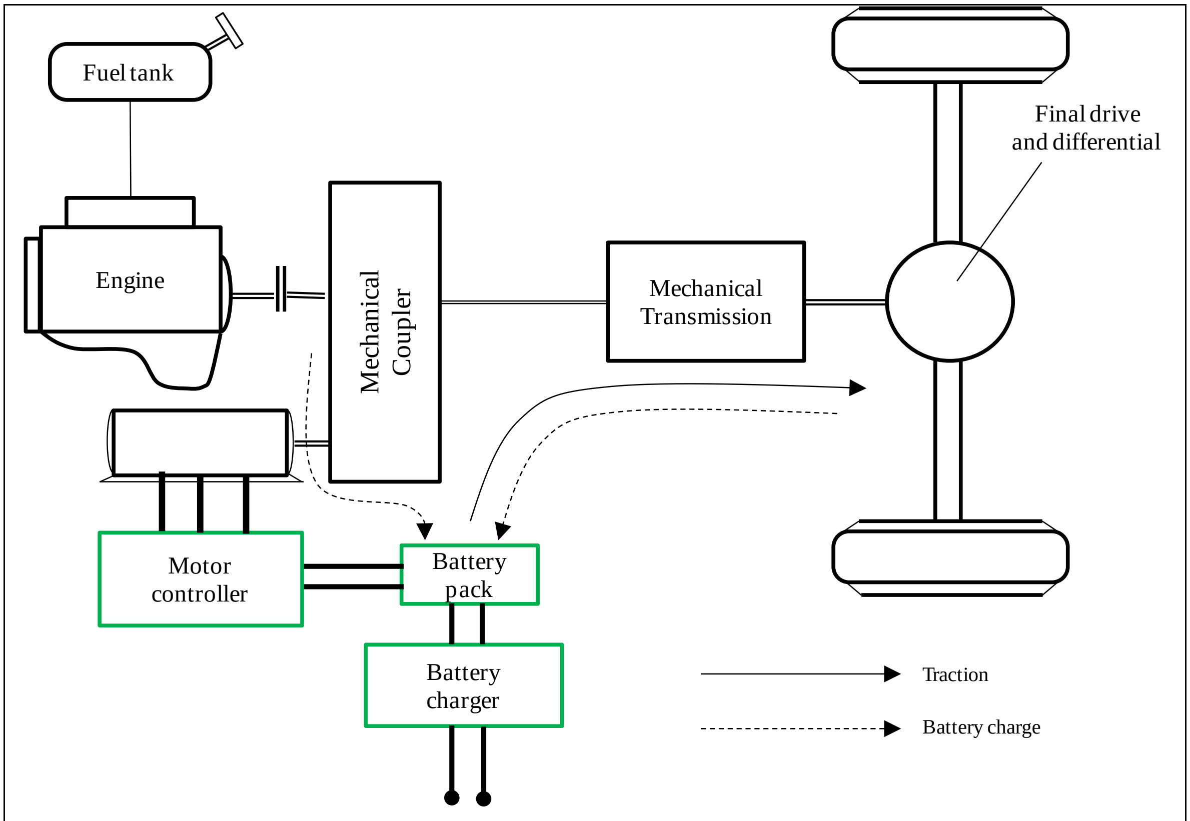

Figure 1: Schematics of a hybrid drivetrain [1]

Table 1: The modes of operation of drive train shown in Figure 1

Operational Mode	ICE Clutch	Clutch	EM	EM Clutch	Battery	Electrical Current	Vehicle Motion
ICE alone Starting	Off -> On	On	Motoring	On	Discharge	Small	At rest
All electric	Off	Open	Motoring	On	Discharge	Big	Moving
ICE and EM Starting	Off -> On	On	Motoring	On	Discharge	Big	Moving
ICE Alone Cruising	On	On	Off	Open	---------	None	Moving
ICE cruising and battery charging	On	On	Generator	On	Charge	Medium	Moving
Acceleration	On	On	Motoring	On	Discharge	Big	Moving
Braking	Off	Open	Generator	On	Charge	Very Big	Moving

Elements of Control Theory

The basis for control is the feedback loop shown in **Figure 2a**. The ***input*** is the desired behaviour of the system and it is compared with the actual behaviour to determine the ***error*** signal. The error signal is fed into one or more control elements that move actuators at the plant. The word "plant" is the name for the object being controlled. The actual value of the controlled variable from the plant is transmitted via the feedback loop to the summation loop.

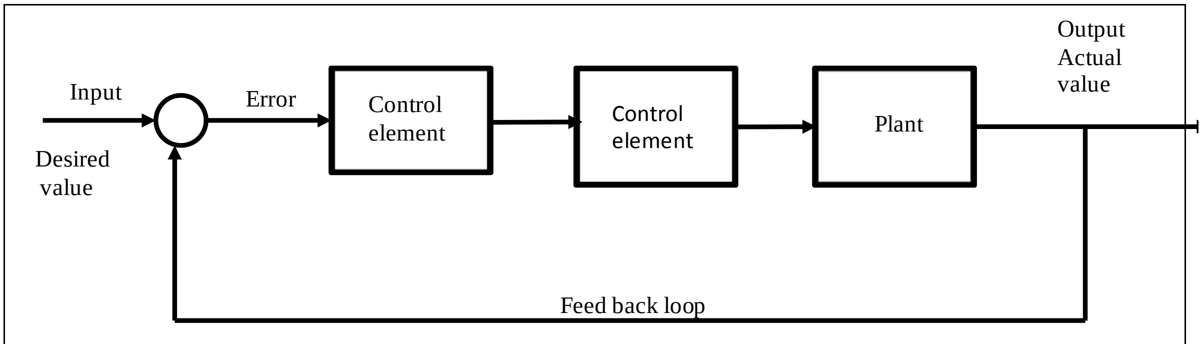

Figure 2a: Basic control system [2]

In **Figure 2b** a simple example of ICE control is shown with its control element and plant. The control element is a general term and identifies the elements that receives the error signal and change the plant's behaviour. In **Figure 2b** the fuel injector is the control element and the ICE is the plant. To understand the working of the control system, shown in **Figure 2b,** let us assume that the desired speed of ICE is 2000 rpm and the actual speed is 2200 rpm. An error signal of -200 rpm is generated and this negative error signal is fed to the fuel injector. The fuel injector responds to the negative error by decreasing the flow of fuel to the ICE. With less fuel, the speed of ICE drops.

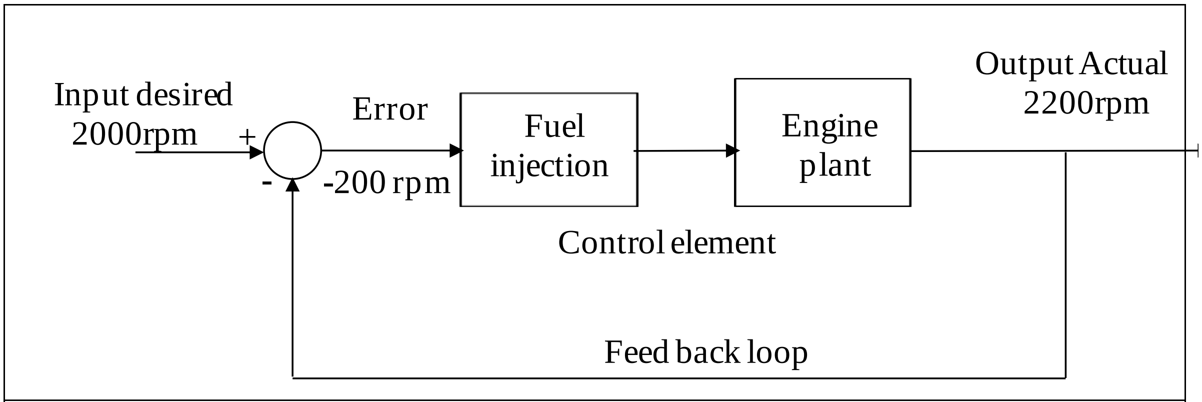

Figure 2b: Basic control system for ICE speed control [2]

A modern control system for HEV may use only a few feedback control loop similar to the one shown in **Figure 2c**. The control architecture shown in **Figure 2c** uses a single feedback loop and has desired velocity as the variable being controlled. The driver is one of the control elements and acts on the error. If the HEV is moving too slowly, the driver steps on the accelerator pedal and it the vehicle is too fast, the driver steps on the brake pedal. The hybrid ECU is the master controller and it controls the other subcomponents of the vehicle such as ICE, EM, power electronics, etc. The detailed discussion on the complete control architecture is discussed in next section.

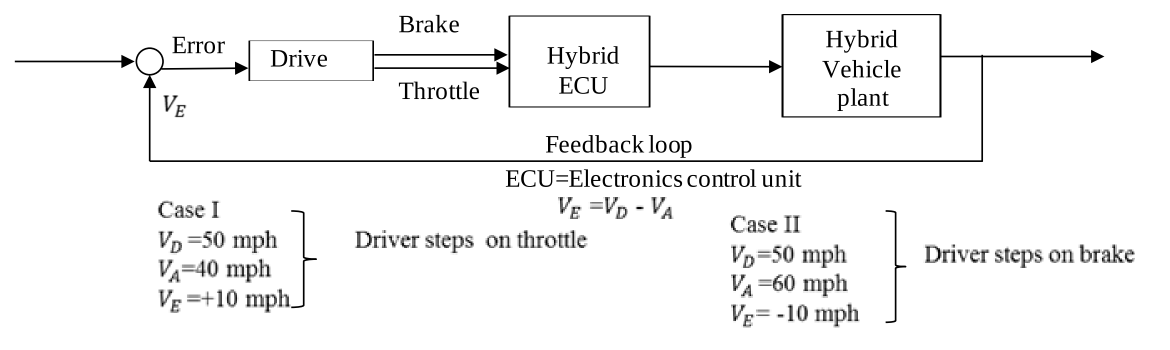

Figure 2c: Control system for vehicle speed control [2]

Overview of Control System: The Electronic Control Unit (ECU)

Typical control architecture of HEV is shown in **Figure 3**. In **Figure 3** it can be seen that there are multiple ECUs such as:

i. Hybrid ECU
ii. ICE ECU
iii. EM ECU
iv. Transmission ECU
v. Power Electronics ECU
vi. Battery ECU or Battery Management System

A brief description of each of the ECUs is given below.

Hybrid ECU**:** The Hybrid ECU is in command of all other ECUs and selects the operational mode based on the driver's input. The hybrid ECU is responsible for system wide energy management. Typically the goal of control is to minimize the fuel consumption. For each litre of petrol, the hybrid ECU tries to provide maximum mileage. To do this, the hybrid ECU allows or prohibits ICE shutoff. The hybrid ECU commands

- the amount of torque and power from the motor and ICE
- the amount and timing of power generation to charge battery.

ICE EMU**:** This controls the various ICE parameters discussed in previous section

EM ECU**:** The EM ECU is responsible for switching of the EM from motoring mode to the generator mode and also controls the motor to deliver the torque demanded by the hybrid ECU. The EM ECU consists of various control strategies such as Constant Torque Control, Field Weakening Control, etc.

Transmission ECU**:** The transmission ECU provides the correct gear ratio to control the torques and angular speeds of the EM and the ICE.

Power Electronics ECU**:** Having power from a battery is only the first step. The power must be delivered to the EM, in the motoring mode, at the voltage and current needed. For regenerative braking, the power must be accepted from the EM. The function of the power electronic ECU is to receive commands from hybrid ECU, to control inverter energy flow both ways, that is, charge and discharge, to control switching of EM between motor and generator modes and to control switching of EM between motor and generator modes.

Battery ECU or Battery Management System**:** The battery ECU or the battery management system (BMS) monitors and measures temperature and assures cooling is adequate. The BMS avoids the stress of heat and over-temperature and the effects of excessive charging or discharging are eliminated or lessened. The BMS is essentially for long battery life and optimum fuel efficiency.

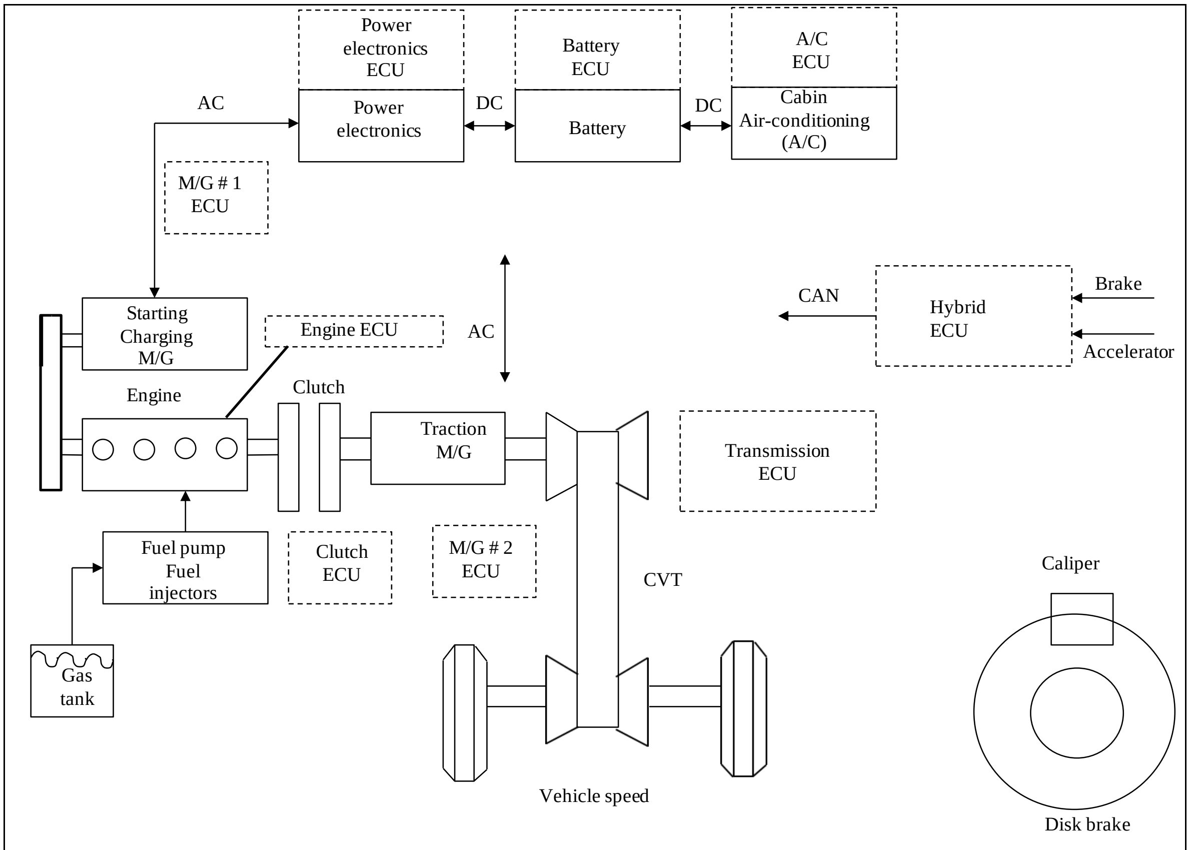

Figure 3: Control architecture of HEV [2]

Control Area Network (CAN)

A typical CAN network in an HEV is shown in **Figure 4**. The CAN is a fast, high rate network enabling communication between ECUs. In CAN most data can be updated every 10ms and the data is checked to assure data reliability.

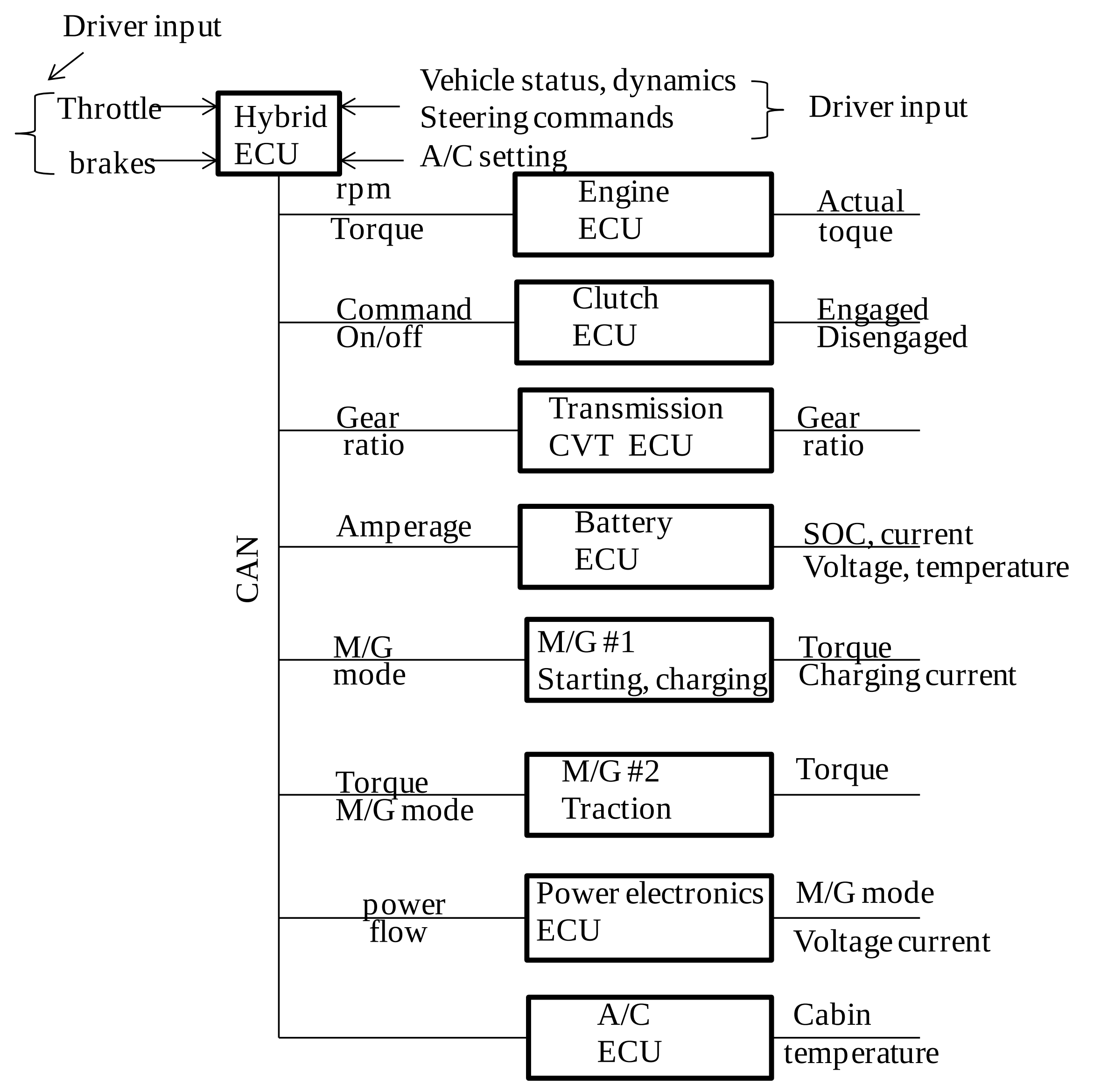

Figure 4: Typical CAN network of a HEVs [2]

Control Variables

The control variables connect various ECUs with each other and fall in one of the three categories:

i. mechanical
ii. electrical
iii. discrete

The control variables falling in the mechanical category are:

- All the variable related to ICE
- Gear ratio
- Rpm of each rotational component

The control variables falling in the electrical category are:

- Currents in the batteries, inverters and EM
- Voltages across EM, inverters and battery terminals
- EM torque

The variables are like yes/no or on/off and HEVs have a few such variables such as:

- EM mode: motor or generator?
- Gear ration: which of the ***n*** available gears?
- Clutch: engaged or disengaged

References:

[1]. M. Ehsani, *Modern Electric, Hybrid Electric and Fuel Cell Vehicles: Fundamentals, Theory and Design*, CRC Press, 2005

[2] A. E. Fuhs, *Hybrid Vehicles and the Future of Personal Transportation*, CRC Press, 2009

Lecture 36: The Hybrid ECU and Its classification

The Hybrid ECU and Its classification

Introduction

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Classification of Hybrid ECU

The hybrid ECU is the heart of the control architecture of any HEV and it is also known energy management strategy (EMS). The EMS can be classified into following broad categories:

i. **Rule based**
ii. **Optimization based**

The ***Rule Based*** strategies consist of following subcategories:

i. **Fuzzy based**: The fuzzy based control strategies are of three types
    a. ***Predictive,***
    b. ***Adaptive***
    c. ***Conventional***
ii. **Deterministic Control**: The deterministic controllers are subdivided into
    a. ***State Machine***
    b. ***Power follower***
    c. ***Thermostat Control***.

The ***Optimization based*** strategies are of following types:

i. **Global Optimization:** The global optimization methods are:
   a. **Linear programming methods**
   b. **Dynamic Programming**
   c. **Stochastic Dynamic Programming**
   d. **Genetic Algorithms**

ii. **Real time Optimization:** The real time optimization techniques are of following types:
   a. **EFC minimization**
   b. **Robust control**
   c. **Model predictive**
   d. **Decoupling Control**

In **Figure 1** the classification tree of the various control techniques is shown. In the subsequent sections the Rule based control strategies will be discussed in detail.

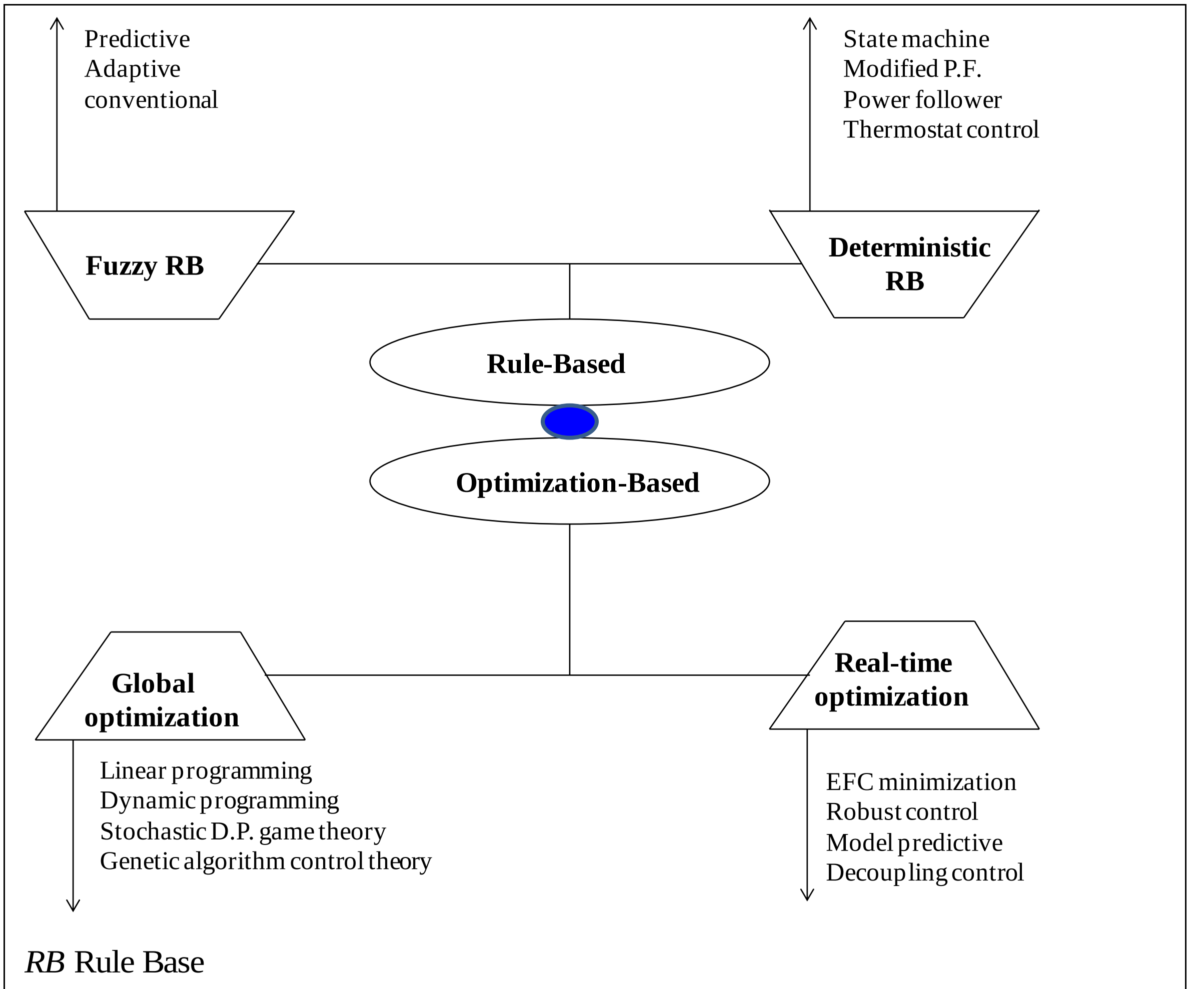

Figure 1: Classification of control strategies [1]

Basic Principles of Rule Based Control Methods

Rule based control strategies can cope with the various operating modes of HEV. The rule based strategies are developed using engineering insight and intuition, analysis of the ICE efficiency charts shown in **Figure 2** and the analysis of electrical component efficiency charts.

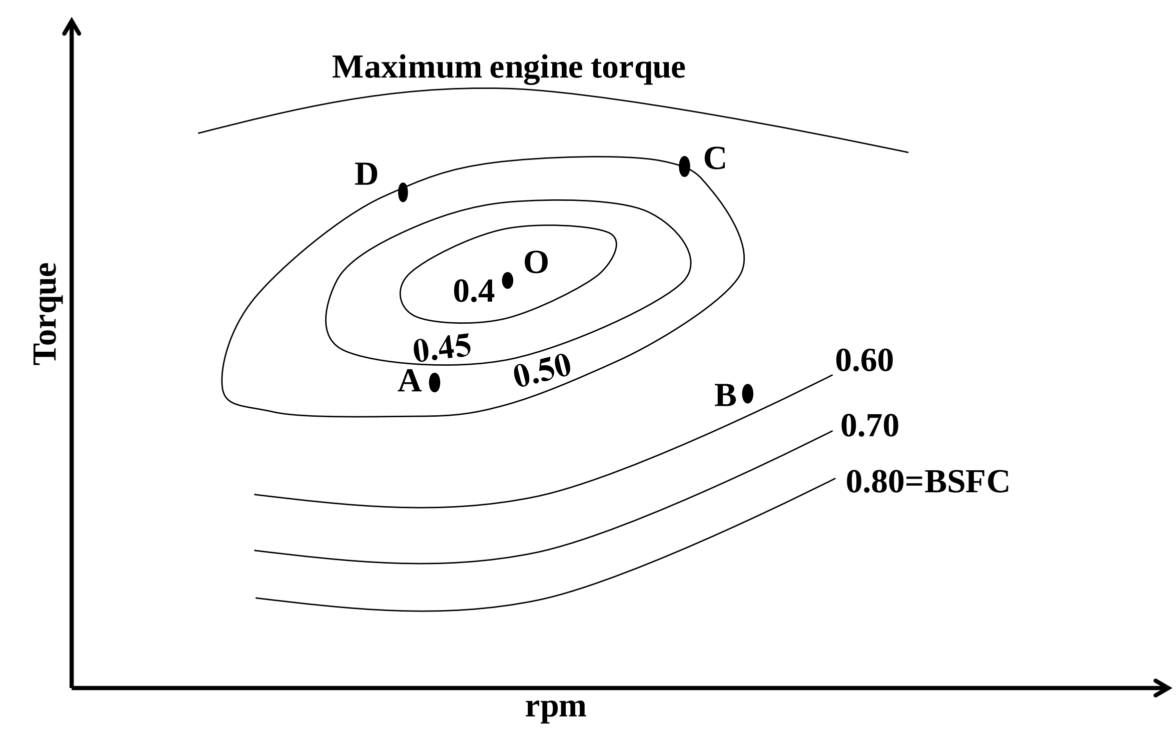

Figure 2: Efficiency map of ICE [1]

An example of developing rule based strategy can be explained using the ICE efficiency map shown in **Figure 3**. The lines, which are drawn using engineering insight and intuition, divide the map into three regions: **A**, **B**, and **C**. The rules for operation of ICE in these three regions are:

i. In the region **A** only EM is used because in this region the fuel efficiency of the ICE is poor.
ii. In region **B** only ICE is used since this the region of high fuel efficiency.
iii. In region **C** both ICE and EM are used.

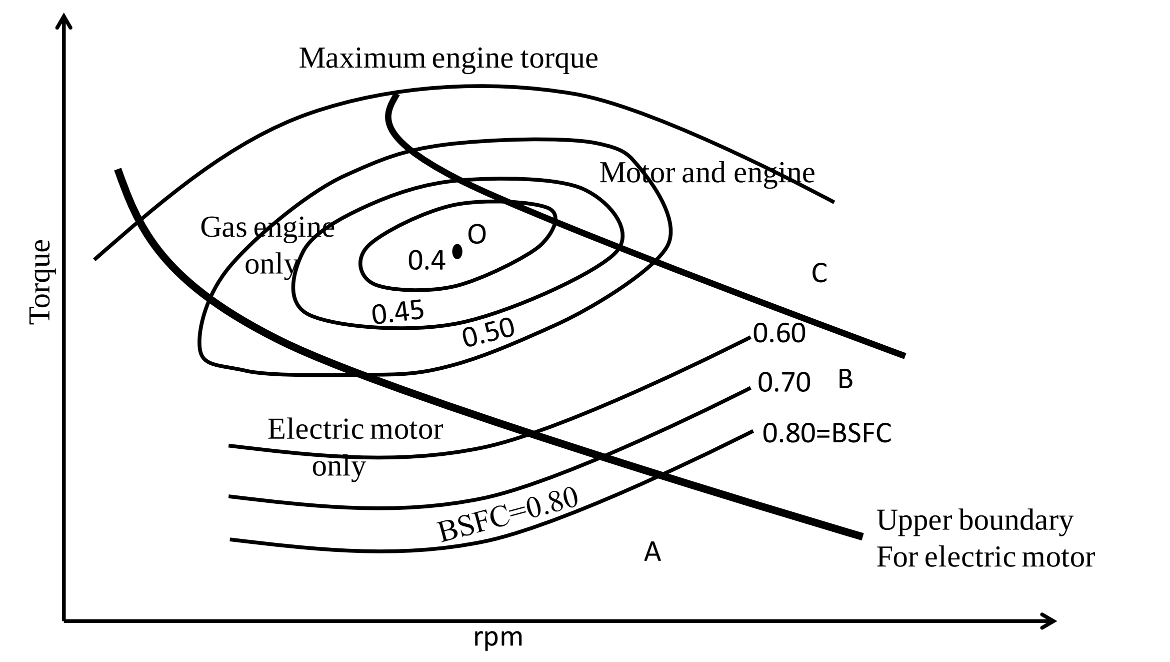

Figure 3: Efficiency map of ICE showing upper and lower boundaries for EM operation [1]

Deterministic Rule Based Strategies

Heuristics based on analysis of power flow in HEV drivetrain, ICE efficiency map and human experiences are utilized to design deterministic rules. These rules are generally implemented using lookup tables to split requested power between the ICE and EM. The most commonly used strategies are:

- Thermostat (on/off) control
- **Power follower control**
- **Modified power follower**
- **State Machine based controller**

In the following sections the controllers marked in bold are explained.

Power follower control

In this strategy the ICE is the primary source of power and the EM is used to provide additional power when needed by the vehicle. Care is always taken to maintain the SOC of batteries within safe limits. The rule base that is generally used is:

i. Below a certain minimum vehicle speed, only the EM is used.
ii. If the demanded power is greater than the maximum power that the ICE can produce at its operating speed, the EM is used to produce excess power.
iii. The EM charges the batteries by regenerative braking.
iv. The ICE shuts off when the power demand falls below a limit at the operating speed. This is done to prevent inefficient operation of ICE.

This is a very simple and effective strategy but the major disadvantage is that the efficiency of the entire drivetrain is not optimized.

Modified power follower

In order to improve the power follower controller a cost function is introduced. The role of this cost function is to strike a balance between fuel consumption and emissions at all operating points of HEV. The rule base for the proposed strategy is as follows:

Define the range of operating points: The range of operating points (distribution of ICE and EM torques) is represented by the range of acceptable motor torques for the current torque request. The relation between the ICE, EM and requested torque is given by

$$T_{ice} = T_{request} - KT_{em} \tag{1}$$

where

$K =$ motor to ICE gear ratio

The greatest possible positive motor torque defines one extreme of the operating point range: This value is the minimum of three values:

a. The driver's torque request
b. The maximum rated positive torque of the motor at the current speed
c. Maximum available positive torque from the EM, according to the limits imposed by the capability of the batteries

The greatest possible negative EM torque defines the other extreme of the operating point range. This value is the maximum of:

a. The difference between the driver's torque request and the maximum positive torque available from the ICE
b. The maximum rated negative torque of the EM at the current speed
c. The maximum available negative torque from the EM, according to limits imposed by the capability of the battery.

For each candidate operating point, calculate the constituent factors for optimization:

The following steps are involved in this step:

a. Calculate the fuel energy that would be consumed by the ICE. The actual fuel energy consumed for a given ICE torque is affected by two things:
   - Hot, steady state ICE fuel maps
   - Temperature correction factors

   For a given torque request and motor torque, **equation 1** sets the ICE torque. At this torque and given speed, the ICE map provides the fuel consumed by the ICE when it is hot (**Figure 4**).

   A cold ICE uses more fuel than a hot ICE. A cold ICE correspondingly produces more emissions than a hot ICE. The outputs of the ICE for cold and hot operation are given by

$$Cold\_use = Hot\_use\left(K_1 + \left(\frac{95 - Temp_{ice}}{75}\right)^{K_2}\right)$$

$$Fuel\_use = Fuel\_hot\left(K_1 + \left(\frac{95 - Temp_{ice}}{75}\right)^{3.1}\right)$$

where (2)

$Cold\_use$: cold consumption ofoutput

Hot_use: hot,steady state variable output

Temp_{ice} : temperature of ICE coolant [°C]

K_1 : a constant that varies with output

K_2 : a constant that varies with output

b. Calculate the effective fuel energy that would be consumed by EM for a time interval, for example 1 second using the following steps:
   - Find fuel energy versus EM torque
   - Find ΔSOC versus EM torque, accounting for gain due to regenerative braking
   - Combine the curves obtained in above steps
   - Determine the equivalent energy by evaluating the curve from step3 at

c. Calculate total energy that would be consumed by the vehicle

d. Calculate the emissions that would be produced by the ICE.

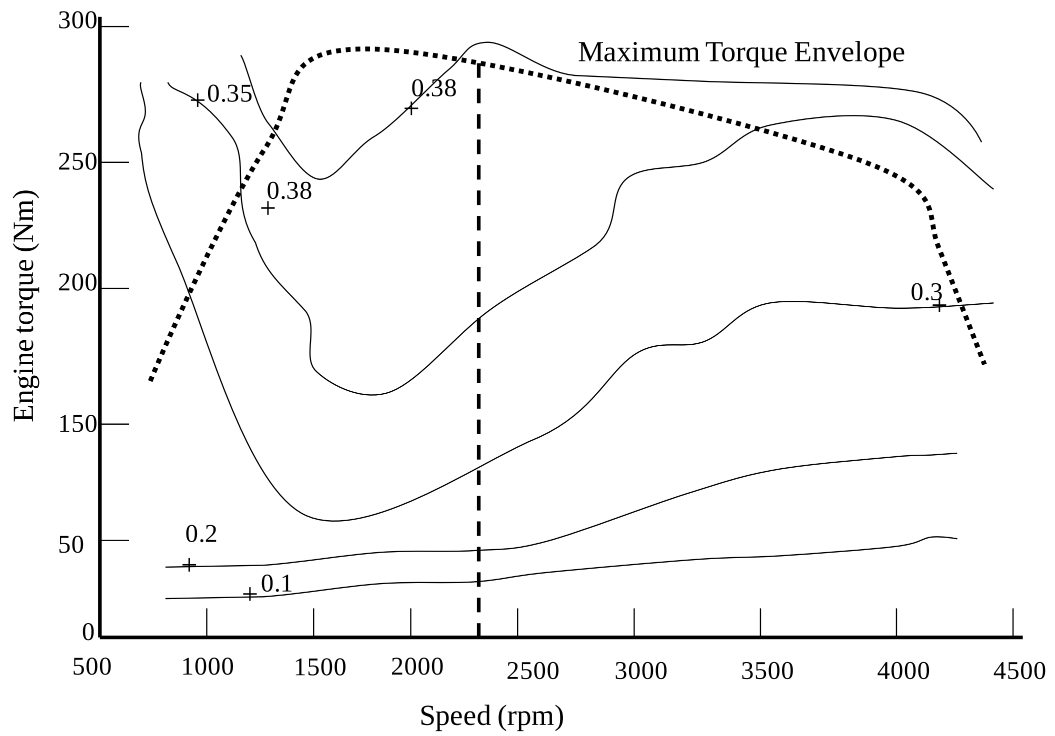

Figure 4: Efficiency map for hot ICE [1]

Normalize the constituent factors for each candidate operating point: The goals of minimizing energy and minimizing emissions can conflict with each other. The most efficient operating point will likely produce more pollution than less efficient operating points. Moreover, minimizing the amount of one pollutant can increase the amount of another. Hence, a second goal of the strategy is to allow prioritization of the relative importance of minimizing the fuel use and each of the pollutants. This prioritization is described in ***Steps 4*** and ***5*** below.

Apply user weighting K_{user} ***to the results from step iii.:*** The relative importance of each of the normalized metrices is determined by two weighing factors. The first is a user weighing fore energy and the emissions. This is basically a Boolean switch for the user to toggle if he/she chooses to ignore certain emissions.

Apply target performance weighting K_{target}: The target performance weighing factor is applied to result from **step iv.** The factor K_{target} is given by

$$K_{\text{target}} = \frac{\text{max of time averaged vehicle performance}}{\text{target performance}} \tag{1}$$

Compute overall impact factor, which is a composite of results of **step iii** to **step v** for all operating points, that is

$$\text{Impact} = \frac{\sum\left(K_{user}^{*} K_{\text{target}}^{*} \text{normalized\_variables}\right)}{\sum\left(K_{user}^{*} K_{\text{target}}^{*}\right)} \tag{2}$$

The flow chart of the control strategy is shown in **Figure 5**.

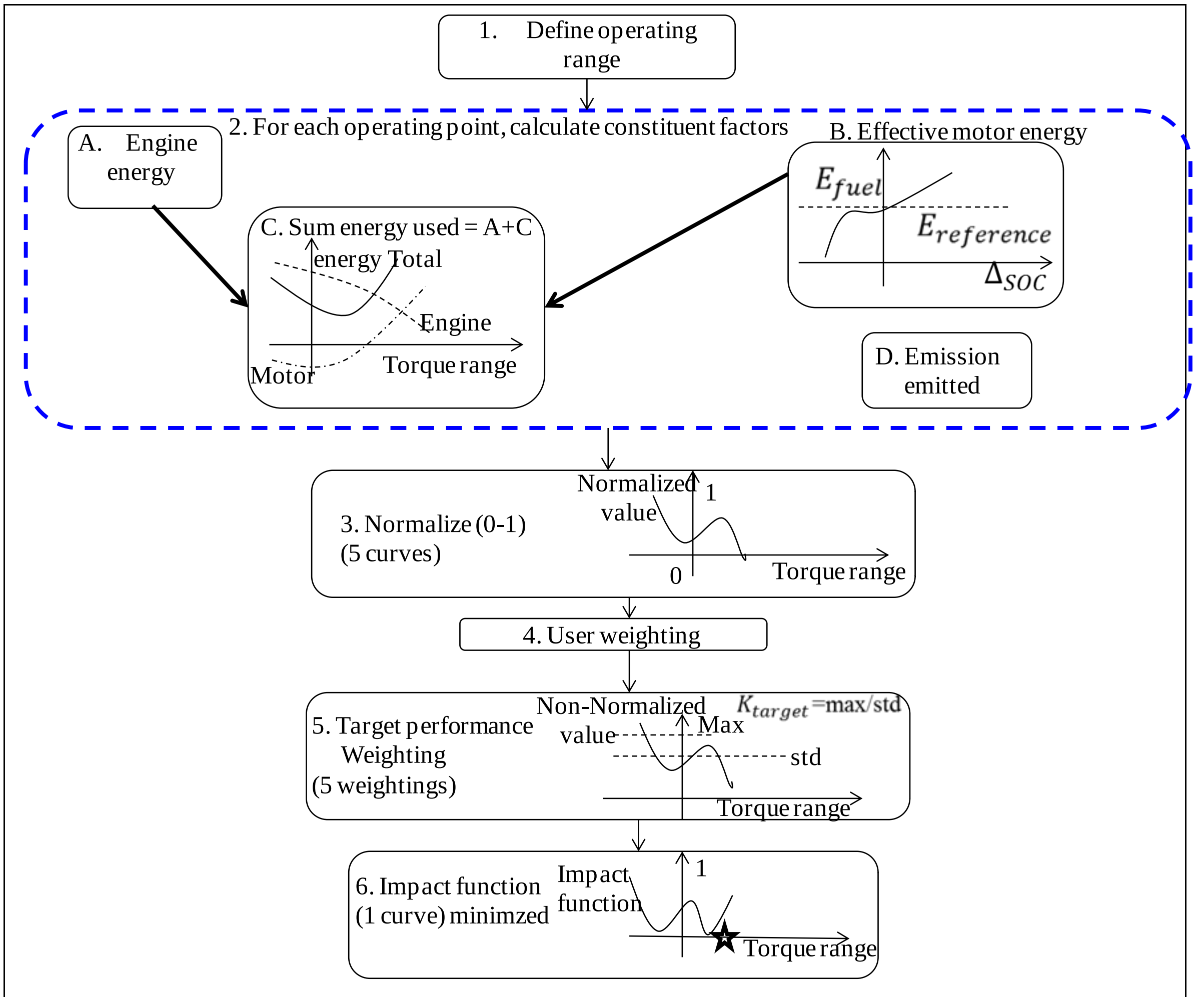

Figure 5: Flow chart for modified power follower controller strategy [1]

The final operating point is the operating point with the minimum impact factor. This strategy improves the overall performance of the HEV drivetrain but is computationally expensive.

State Machine Based

The state machine dictates the operating mode of the HEV such:

i. ENGINE (ICE propelling the vehicle)

ii. BOOSTING (both ICE and EM propelling the vehicle)

iii. CHARGING (ICE propelling the vehicle and charging the battery)

The transition between the operating modes is decided based on:

i. the change in driver demand

ii. a change in vehicle operating condition

iii. a system or a subsystem fault.

The various states involved in the control strategy are listed in **Table 1**.

Table 1: States of an HEV

State	**ICE**	**Clutch**	**EM**	**Description**
Off	Off	Disengaged	Off	Vehicle off state
EM drive	Off	Disengaged	Motoring	EM propels the vehicle
Regeneration – Low velocity	Off	Disengaged	Generating	Regenerative Braking with ICE disconnected
Regeneration – High velocity	Off	Engaged	Generating	Regenerative Braking with ICE connected
ICE drive	On	Engaged	Off	ICE propelling the vehicle
Boost	On	Engaged	Motoring	ICE and EM propel the vehicle
Charging	On	Engaged	Generating	ICE propels the vehicle and charges the batteries
ICE Stop	Off	Disengaged	Motoring	Motor propelling the vehicle and ICE disconnected
ICE Start	On	Engaged	Motoring	Motor propelling the vehicle and starting the ICE
Bleed	On	Engaged	Motoring	ICE propelling the vehicle and motor discharging the battery

Implementation of a vehicle controller through state machines facilitates fault resilient supervisory control of the whole system.

References:

[1] A. E. Fuhs, *Hybrid Vehicles and the Future of Personal Transportation*, CRC Press, 2009

Lecture 37: The Fuzzy Logic Based Control System

The Fuzzy Logic Based Control System

Introduction

The topics covered in this chapter are as follows:

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Why Fuggy Logic (FLC) Based Controllers for HEVs

Looking into a hybrid drivetrain as a multidomain, nonlinear and time varying plant, fuzzy logic seems to be the most logical approach to the problem. Instead of using deterministic rules, the decision making property of fuzzy logic can be adopted to realize a real time and suboptimal power split.

Fuzzy logic is an extension of the conventional rule-based methods and has following advantages over them:

- ***Robustness*****:** It is inherently robust because it does not require precise, noise free inputs and the output is a smooth function despite a wide range of input variations.
- ***Adaptation*****:** Since FLC processes user defined rules governing the system, it can be modified easily to improve or drastically alter system performance.
- ***Flexibility*****:** FLC is not limited to a few feedback inputs and one or two outputs and it is not necessary to measure or compute rate-of-change of parameters.

How to Use Fuzzy Logic in Design of Controllers

The energy management and control strategy using FLC performs following actions:

- maximizes fuel economy, minimize emissions and distribute the driver's request for power between two sources: ***ICE*** and ***Motor***.
- maximize fuel economy at any point in operation, that is, provide dynamic or instantaneous optimization.
- maximize some other attributes such as acceleration of vehicle

In **Figure 1** the generic schematic diagram of FLC is shown. The four components of FLC, as shown in **Figure 1**, are:

- Fuzzification, which is the change from crisp values to fuzzy values. For ICE speed, the crisp value may be 2000rpm and a possible rule for ICE speed may be "If ICE rpm is too low, then inject more fuel". The fuzzy value associated with the "If X, then Z" statement would be for X<2000 rpm.
- Rule base has a collection of rules: several hundred rules may be developed and applied.
- Inference applies the defined rules to the inputs.
- Defuzzification transforms the results of the inference process to crisp outputs.

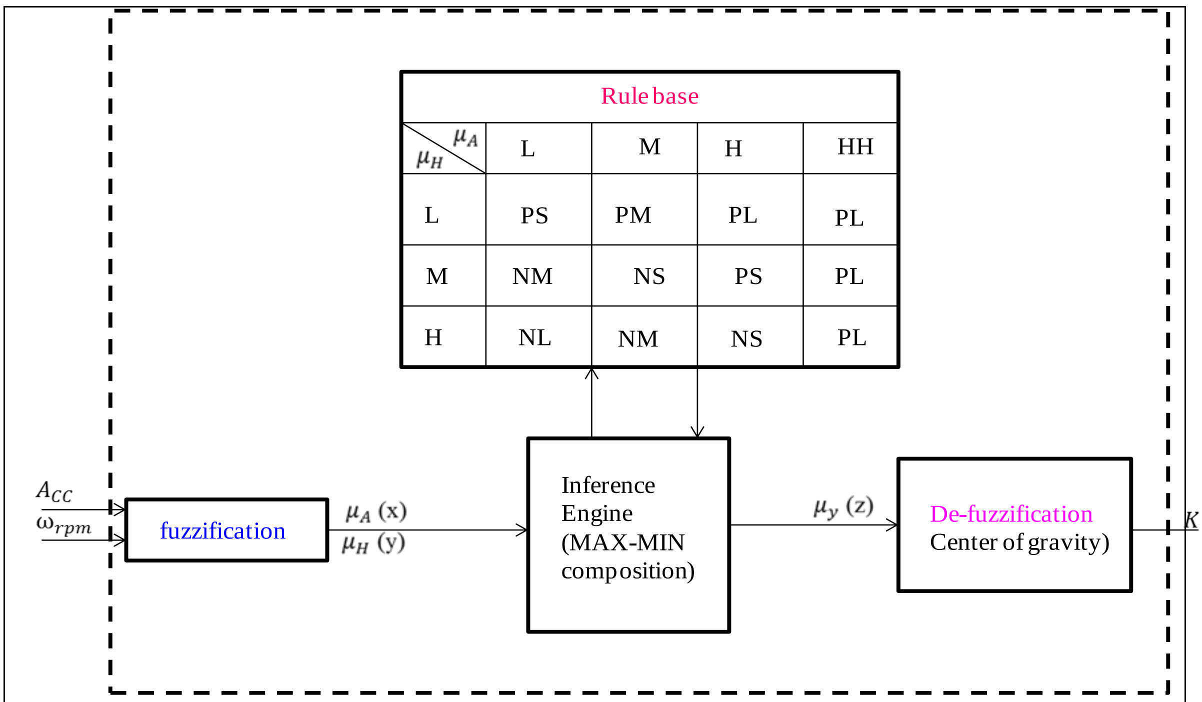

Fig.1: Generic block diagram of FLC [1]

Fuzzy Strategy

The FLC, explained in this section, satisfies the following objectives:

- minimize NOx emissions
- sustain battery SOC
- achieve desired torque requested by the driver

The inputs to this FLC are:

- Acceleration pedal stroke (Acc)
- EM speed (ω_{em})

The configuration of the drive train is shown in **Figure 2**. An induction motor (IM) used in the drivetrain and the IM is directly coupled to the ICE. Since the IM is directly coupled to the diesel ICE, it will be in the field weakening region in most of the ICE operating, the generating torque decreases as the ICE speed increases.

Hence, it is required to describe the required torque as a ratio defined as K to the rated torque at a rotational speed. The positive K means that the IM acts as a powering source and negative K means that the IM acts as a generator. Once K is determined, the torque command becomes

$$\text{Torque Command} = K \times \text{rated torque at a rotational speed} \tag{1}$$

Some basic principles of generating the torque command from the acceleration pedal stroke and ICE rotational speed in the HEV can be described as follows:

- ***Low ICE Speed:*** When the ICE rotational speed is low, it generates pollutant emissions with low efficiency. Hence, in this operating condition, the torque assistant control by the IM should be performed. Assistant torque is commanded to increase in proportion to the acceleration pedal stroke as in the conventional ICE vehicle, that is:

  $$K \times \text{acceleration pedal stroke} \tag{2}$$

- ***Medium ICE Speed:*** When the diesel ICE's speed is medium, it can supply sufficient torque to the hybrid drive train. Hence, battery recharging control is performed instead of torque assistance control when acceleration pedal stroke is below some extent. The torque assistance of the IM should be should be achieved to satisfy the driver's acceleration need if the acceleration pedal is pressed beyond some extent. Since, the diesel ICE torque is subjected to saturation beyond 80% of the pedal acceleration pedal stroke; the motoring action of the IM is made to begin from that point.
- ***High ICE Speed:*** When the diesel ICE speed is high, the torque assistance control is performed as that of the medium speed range. In battery recharging control the IM's output power is kept constant. Now since the ICE can produce more power than in the medium speed range, the factor K should be made to be negatively greater in order to supply more power to the batteries than that of the medium speed range. As the speed increases, the ratio of the power capability of the ICE to that of the IM increases. Hence, it is beneficial to recharge the battery at high speed, rather than at medium speed.

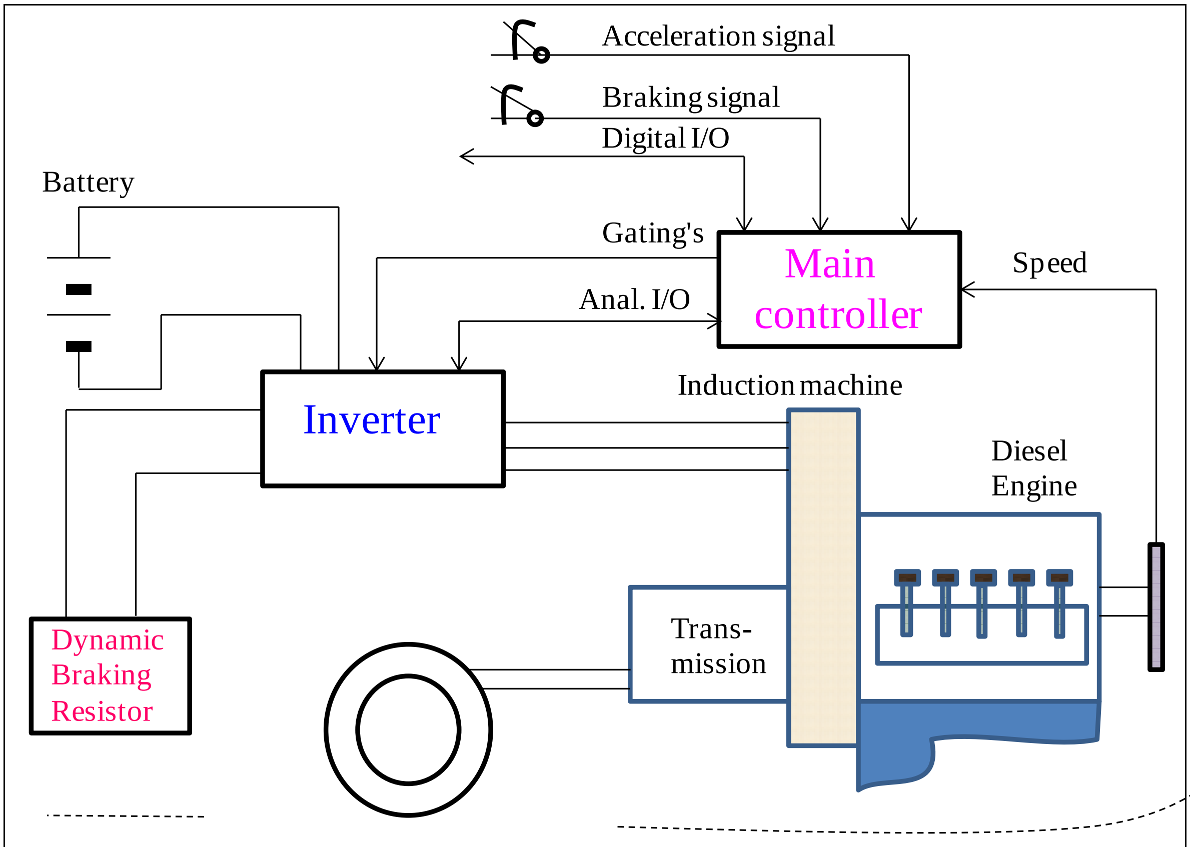

Fig.2: Drivetrain configuration [1]

Implementation of Fuzzy Logic

Based on the three principled of operation of ICE, discussed above, the fuzzy rule base can be developed. The development of the FLC is described in the following subsections.

Input / Output Membership Functions for Fuzzy Logic

For the considered example, there are two input variables namely:

- The acceleration pedal stroke Acc
- The IM rotational speed ω_{rpm}

The ranges of the input variables are set as follows:

- The Acc is set to zero when the driver does not press the acceleration pedal at all and set to a 100 when the driver presses the acceleration pedal completely.
- The ω_{rpm} can vary from the diesel ICE's idling speed to its maximum speed.

The output is the normalized ratio of the torque command to rated torque at a speed. The inputs and the outputs are normalized between zero and one. The input and the output membership functions are shown in **Figure 3**.

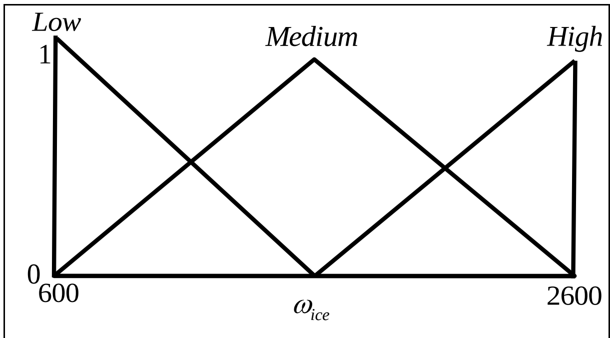

Figure 3a: Membership functions for ω_{ice}

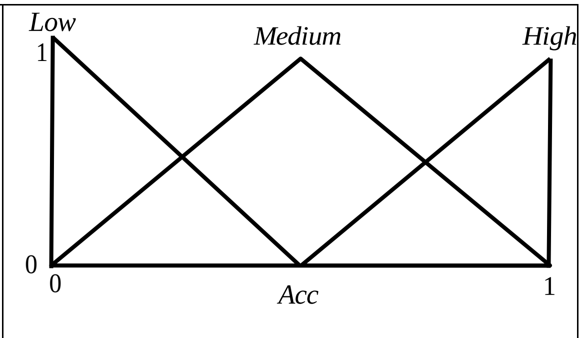

Figure 3b: Membership functions for Acc

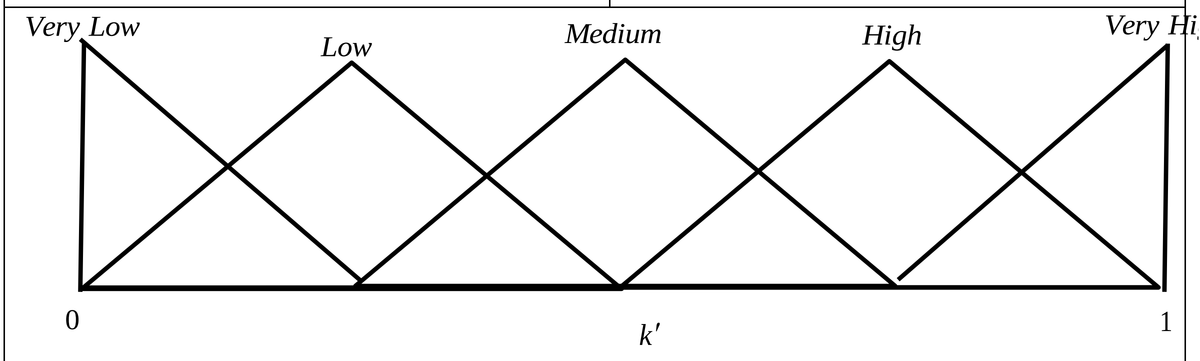

Figure 3c: Membership functions for K'

Rule Base for Fuzzy Logic

The rule base for the torque control and battery recharging control are given in **Table 1**.

Table 1: Rule base

Input 1 (ω_{ice})	**Input 2 (Acc)**	**Output (K')**
Low	Low	Medium
Low	Medium	Low
Low	High	Very Low
Medium	Low	High
Medium	Medium	Medium
Medium	High	Low
High	Low	Very High
High	Medium	Medium
High	High	Low

Procedures and Results of Fuzzy Logic

In fuzzificiation process, the normalized crisp inputs x for Acc and y for ω_{rpm} are transformed to fuzzy values in singletons.

The singletons are fuzzy sets which have a membership's value of 1 for the given inputs and 0 for all other values:

$$\mu_{A_o}(x) = \begin{cases} 1 \text{ if } x = x_o \\ 0 \text{ if } x \neq x_o \end{cases} \tag{3}$$

$$\mu_{B_o}(y) = \begin{cases} 1 \text{ if } \mathrm{y} = y_o \\ 0 \text{ if } \mathrm{y} \neq y_o \end{cases} \tag{4}$$

where x_o and y_o are normalized operating points for Acc and ω_{rpm} respectively. The truth values for the i^{th} input membership function for x and the j^{th} input membership function for y, ω_{ij} can be obtained as

$$\omega_{ij} = \min\left\{\mu_{A_i}(x), \mu_{B_j}(x)\right\} \text{ where i=1,2,3; j=1,2,3} \tag{5}$$

Using ω_{ij} and k^{th} output membership function according to each rule, $\mu_{C_k}(z)$, fuzzy output value, $\mu_{C_{ij}}(z)$ can be calculated as

$$\mu_{C_{ij}}(z) = \min\left\{\mu_{ij}, \mu_{C_k}(z)\right\} \text{ where k=1,2,3,4,5} \tag{6}$$

Finally, the fuzzy set for output z, $\mu_{co}(z)$ can be calculated using the union operator

$$\mu_{CO}(z) = \max\left\{\mu_{C11}(z), \mu_{C12}(z), ..., \mu_{C42}(z), \mu_{C43}(z)\right\} \tag{7}$$

Inference results are transformed into crisp value through centre of gravity method

$$K' = \frac{\int \mu_{CO}(z).zdz}{\int \mu_{CO}(z)dz} \tag{8}$$

K' varies from 0 to 1.

Example

The above calculations are explained using an example. The example is illustrated in **Figure 4**. The following can be observed from **Figure 4:**

- The entire rule base is shown in form of triangular membership functions
- Each triangle has a height of 1

Let us assume the following input:

- $\omega_{ice} = 1600$ rpm
- $Acc = 0.5$

Now, from the figure it can be seen that the value of 1600 rpm belongs to ***Medium*** membership for input ω_{ice} function and the value of 0.5 belongs to ***Medium*** membership function for the input Acc. Hence, from **equation 3**

- $\mu_{A_o}(1600) = 1$ for Medium
- $\mu_{B_o}(0.5) = 1$ for Medium

The **equation 3** gives $\omega_{ij} = \min\left\{Medium, Medium\right\}$ =Medium .

From **Table 1** it can be seen that for the given inputs, the **rule 5** is activated. Hence, **equation 7** gives . Using the **equation 8**, the centre of gravity is obtained as 0.5, that is $K' = 0.5$.

References:

[1] A. E. Fuhs, *Hybrid Vehicles and the Future of Personal Transportation*, CRC Press, 2009

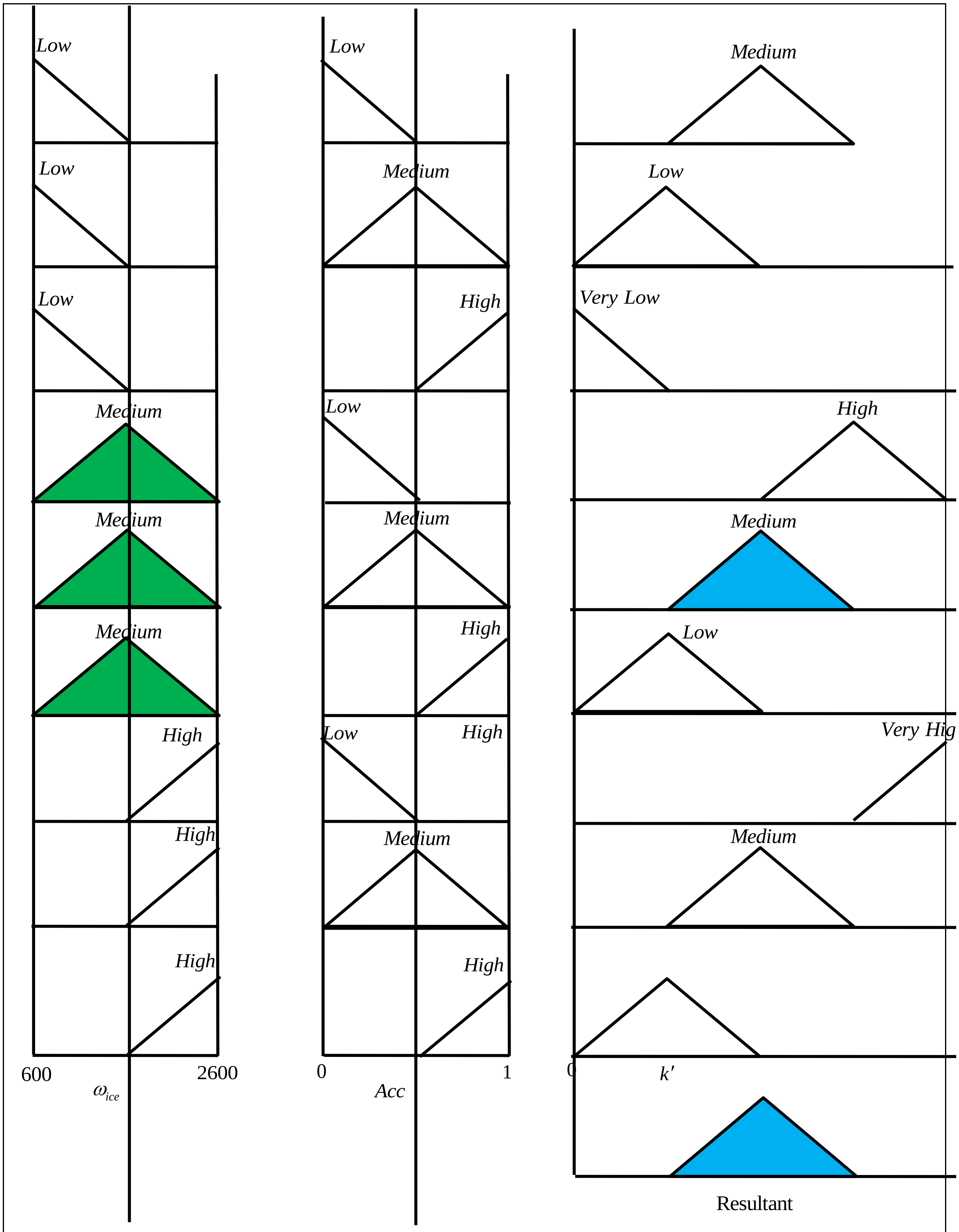

Figure 4: Graphical representation of rule base